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ERRATA

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Page 245, line 9, for "Real Hermitian" read "Hermitian".

Page 248, line 3, for "$f^{(r-1)}(\lambda)$ must equal $\alpha(\lambda)$" read "$\alpha(\lambda)$ is a factor of $f^{(i)}(\lambda)$ $(i = 0, 1, \cdots, r-1)$".

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Page 297, Omit all subscripts in (3.4), (3.5), (5.1).

Page 297, line 4 from foot, for "The ith component etc." read "This solution is $\int_a^b G\cdot\beta\, dt + u$".

Page 299, In (4.6), (4.7) interchange the subscripts of K.

Page 299, line 8 from foot, omit subscripts.

ON THE PROBLEM OF COLORING MAPS IN FOUR COLORS, I.*

By C. N. Reynolds, Jr.

Introduction. The problem of coloring in four colors the map of a simply connected closed surface has been reduced to the problem of coloring maps in which certain configurations, known as reducible configurations, are absent.† In this paper we shall develope some methods of so analysing the known geometric reductions of our problem as to discover and to prove some of their more important implications.

We shall use the reductions due to Kempe, Birkhoff and Franklin together with Errera's reduction of pairs of adjacent pentagons surrounded by hexagons. The reductions of Kempe, Birkhoff and Franklin are sufficient hypotheses for our final theorem that any irreducible map has at least twenty-eight regions. The use of one of Errera's reductions simplifies the proof of that theorem by eliminating much of the case making which would otherwise be necessary.

Our fundamental method will be a systematic study of a set of geometric operations which suffice to build any connected configuration of pentagons which may exist in an irreducible map. Under these operations certain numerical topological characteristics are found to undergo well defined increments. Linear relations between these increments imply homogeneous linear difference equations which yield certain homogeneous relations between our topological characteristics. These relations were originally found inductively from a study of particular configurations.

Secondly we shall prove synthetically certain inequalities between the topological characteristics of an irreducible map, making use of the linear relations mentioned above to reduce them to more serviceable forms.

We shall then apply these inequalities to the proof of the theorem that any irreducible map must have at least twenty-eight regions.

Finally we shall describe: (a) A map of twenty-eight regions which is irreducible with respect to the reductions of Kempe, Birkhoff and Franklin, but reducible with respect to the reduction due to Errera which we mentioned above. This map proves that what we may call the pre-Errera

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* Presented, in part, to the American Mathematical Society, Feb. 28, 1925.

† cf. Franklin, *The four color problem*, Amer. Journ. of Math., 44 (1922), p. 42; also A. Errera, *Une contribution au problème des quatre couleurs*, Bull. Soc. Math. de France, 53, (1925), p. 42. Most of our references will be to these two papers.